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Estimation of the random incidence sound absorption coefficients of different size rectangular samples

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ABSTRACT:
The sound absorption coefficient (SAC) of materials measured in a reverberation room is affected by both the intrinsic properties of the material and geometrical dimensions of the sample. A different size of the same material may produce a different SAC primarily due to the edge effect phenomenon. In this research, the experimental data from multiple laboratories was analyzed to evaluate the influence of the edge effect. An empirical function was established based on these measurement data and the linear relationship between the SAC and the relative edge length. Thomasson's method, the two geometric methods, and the analytical method were used to estimate the SAC of an absorber from measurements on a different size sample and compared with results obtained using the empirical function. The results show that the proposed empirical method is a reliable way to predict the SAC of a sample from measurements on a different size sample of the same material, which only requires the thickness, density, and size of the material. https://doi.org/10.1121/10.0022384

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I. INTRODUCTION

The sound absorption coefficient (SAC), as measured in a reverberation room, depends not only on the material properties but also depends on the geometrical properties of the tested sample. A well-known observation is the measurement result of the SAC in a reverberation room, where it exceeds 1. Furthermore, varying the size of samples made of the same material can result in substantial differences in SAC measurements, especially for materials with high sound absorption. This occurrence is referred to as the "edge effect" and was discovered by Sabin in the early 1900s. In theory, it is caused by the diffraction of the incident wave at the discontinuity in the imaginary part of the specific acoustic admittance at the edges of the sample. Therefore, the sound pressure incident on the sample is greater due to the sudden change of the imaginary part of the specific acoustic admittance at the edges, and this increase leads to additional absorption near the edges of the sample. According to the early literature,[1–7] there is a linear relationship between the SAC obtained by the reverberation room method and the sample's size as shown in the following equation:

$$\alpha = \alpha_0 + \beta E, \tag{1}$$

where α is the SAC of a finite size sample measured in a reverberation room, α_0 is the true absorption coefficient whose value may be equal to the absorption coefficient of an infinite-size sample, β is a constant, and E is the sample's relative edge length which is the ratio of the perimeter divided by the area of the sample. The details of the process of using linear regression to obtain the constants is shown in Fig. 1. Measurement of four different size samples from the identical material allows one to graph SACs as a function of the relative edge length and fit them with linear regression lines, where β is the slope of the linear regression and α_0 is the constant of the linear regression.

Since the early 1900s, numerous theoretical and experimental studies have been conducted on the edge effect phenomenon. A theoretical study by Morse and Bolt[8] showed that, for normally incident sound, the additional absorption caused by sound diffraction is approximately equal to the product of the quarter wavelength of the sound, the edge length of the specimen and the difference of the acoustic susceptance of the material and the acoustic susceptance of the surface surrounding the sample. Cook[7] plotted the SAC as a function of the area from experimental data to fit the linear regression of Eq. (1) and calculated the values of β and the corresponding α_0. Similar research was performed by Daniel,[3] where in addition to proving the linear relationship, he measured the distribution of sound pressure at the surface of a square panel and suggested that the additional absorption is proportional to the inverse of the square root of the sample's area. Ten Wolde[6] verified the linear relationship by comparing the α_0 obtained from impedance tube measurements with those extrapolated from reverberation room measurements. This research confirmed that the

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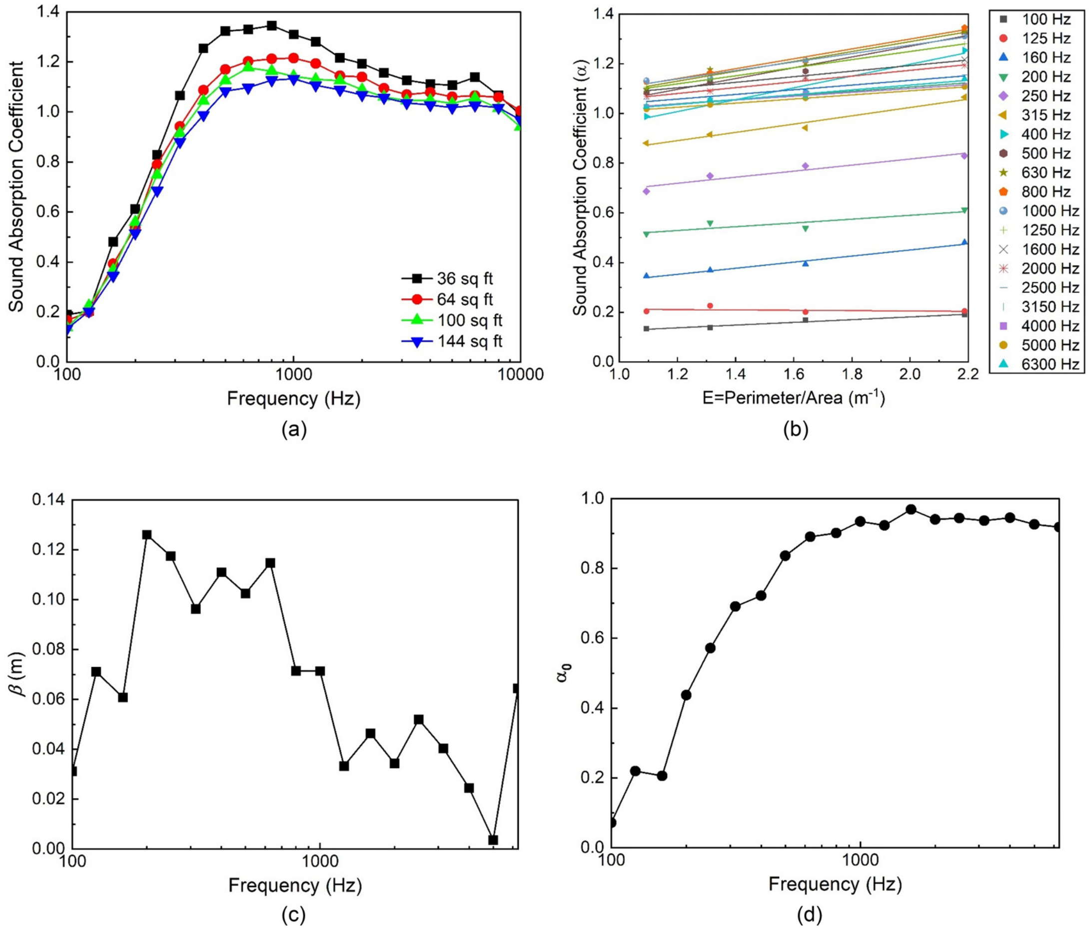

FIG. 1. (Color online) (a) The measured SAC of the same material with different sizes. (b) Relationship between the SAC and the relative edge length, with linear regression lines for fitting. (c) The slope of the linear regression. (d) The constant of the linear regression.

extrapolation of the linear relationship can be used to estimate the true absorption coefficient α_0. Bruijn[9] tried to explain the linear relationship by using a mathematical model. He derived a formulation to examine the interaction between the two local edge diffraction fields of an acoustic strip and he found that the additional absorption heavily depends on the sample's geometry for small patches. Bartel[10] compared experimental data with theoretical data of the random incidence SAC, confirming that both results increase linearly with the relative edge length. More researchers including Kosten,[1] Khul,[2] Gomperts,[4] Esche,[5] Dekker,[11] Vercammen and Lautenbach,[12] and Hughes *et al.*[13] observed the linear relationship by plotting the SAC data as a function of the relative edge length of the specimen. Despite numerous researchers calculating β and α_0, few have put them into practical use. In fact, this linear relationship can be utilized to estimate the random incidence SAC of samples with different sizes of the same material. It might act as a complement to the ISO 354 standard,[14] which imposes restrictions on the size of the test sample.

Several methods have been established to put the theory of edge effect into practice. Thomasson[15] predicted the SAC of one size sample from other size samples of the same material by introducing the maximum SAC that only depends on the sample's geometrical properties. It means that the SAC of different shapes of the same material can be determined by an equivalent of the maximum SAC and the SAC of an infinite-size sample can also be calculated. The draft version of the new ISO 354 standard incorporates Thomasson's approach and proposes a simpler method for estimation; however, this draft was ultimately rejected in spring of 2023. It assumes the constant β is one-quarter of the wavelength ($\frac{1}{4}\lambda$), which means that the width of the extended virtual surface along the edge (the area of additional absorption) is equal to the value of $\frac{1}{4}\lambda$. A comparison of $\frac{1}{4}\lambda$ with the value calculated from Thomasson's method was made by Zhao *et al.*,[16] who extended Thomasson's theory to the non-locally reacting material, which indicated that $\frac{1}{4}\lambda$ is not correct for low SACs. Additionally, they modified Thomasson's linear interpolation method to improve the accuracy of predicting the SAC of a large size sample from a small size sample. It also gives corrections which are different for increasing and decreasing sizes. While their approximate analytical approach does decrease the computation time, it still has some disadvantages such as complicated formulas and limited available material experimental data. Sauro *et al.*[17] plotted the absorption divided by the perimeter as a function of the area divided by the perimeter and fitted to data points using linear regression line for each one-third octave band. They presented a new equation to estimate the sound absorption of a material of any size, but their approach requires at least two (and preferably more) different sample sizes to be measured in a reverberation room initially. While this is the preferred method if reverberation measurements on multiple size samples are available, the aim of this paper is to develop a method using a measurement on only one size of sample whose correction is independent of whether the sample size is being increased or decreased. Other literature related to the edge effect includes enhancing the theoretical prediction of the random incidence SAC for sound-absorbing materials or making measurements on a small sample group of theatre chairs.[18–20] Despite this, there is still room for further examination and study. Based on the discussions presented, the implementation of the theory for practical applications remains challenging.

Given this context, the goal of this study is to formulate a straightforward empirical equation for calculating the random incidence SAC of a material for different sizes of samples. The critical aspect of this equation is determining an average value of the constant β which varies between materials due to its connection to material properties. Consequently, the research described in this paper is commenced by collecting all available data on the SAC of different size samples of the same material in the literature for 24 materials and different laboratories to investigate the characteristics of the relative edge length multiplier β and β/λ. In addition, an empirical function is proposed utilizing all the collected experimental data, which only requires the thickness and density of the material. It allows one to predict the SAC of a sample of one size from measurements on another size of the same sound absorbing material without repeated measurements or complicated calculations. None of the other material properties, such as the acoustic impedance or the six parameters of the Johnson-Champoux-Allard-Lafarge model[21] are needed. The feasibility of the proposed method is verified by comparing it with other estimation methods including Thomasson's method, the two geometric methods, and the analytical method proposed by Zhao *et al.*[16]

This paper begins by describing the theory of the edge effect estimation methods including linear relationship method, Thomasson's method, and geometric method in Sec. II. The database of materials and data analysis is described in Sec. III. Section IV presents the development of the empirical equation and Sec. V compares the results from the proposed empirical function with the results from the other estimation methods. Finally, some further suggestions are made.

II. METHODOLOGY

A. Linear relationship method

An equation for calculating the sound absorption of one size of sample from the measured sound absorption of two other sizes of samples of the same materials was derived by Sauro *et al.*,[17]

$$A_x = \left(\frac{\dfrac{A_2}{P_2} - \dfrac{A_1}{P_1}}{\dfrac{S_2}{P_2} - \dfrac{S_1}{P_1}}\right) \times S_x + \left[\frac{A_1}{P_1} - \left(\frac{\dfrac{A_2}{P_2} - \dfrac{A_1}{P_1}}{\dfrac{S_2}{P_2} - \dfrac{S_1}{P_1}}\right) \times \frac{S_1}{P_1}\right] \times P_x, \tag{2}$$

where A is the sound absorption, S is the area, and P is the perimeter. The subscript x denotes the sample size whose sound absorption is being calculated, and subscript 1 and subscript 2 denotes the two different sample sizes which have been measured in a reverberation room. In this study, the authors rewrite Eq. (2). First, divide both sides by S_x, and the equation becomes

$$\frac{A_x}{S_x} = C + \left(\frac{A_1}{P_1} - C \times \frac{S_1}{P_1}\right) \times \frac{P_x}{S_x}, \tag{3}$$

where

$$C = \frac{\dfrac{A_2}{P_2} - \dfrac{A_1}{P_1}}{\dfrac{S_2}{P_2} - \dfrac{S_1}{P_1}}. \tag{4}$$

Then, make

$$k = \frac{A_1}{P_1} - C \times \frac{S_1}{P_1}. \tag{5}$$

Equation (3) can be written as

$$\frac{A_x}{S_x} = C + k \times \frac{P_x}{S_x}. \tag{6}$$

It is obvious that the Eq. (6) is similar with Eq. (1) because A_x/S_x is the SAC to be calculated and P_x/S_x is the relative edge length. Consequently, as long as the SACs and the geometry of two different sample sizes of the material are known, it is possible to estimate the SAC of other size samples of the same material. Looking at Eq. (1), once the constant β is known, the SAC of other size samples can be estimated from the SAC of one size sample as follows:

$$\alpha_x = (\alpha - \beta E) + \beta E_x = \alpha + \beta(E_x - E), \tag{7}$$

where α_x is the SAC of the size of sample to be predicted, and its relative edge length is E_x. α is the SAC of a sample measured in the reverberation room with the relative edge length E. The width of the area subject to the increased pressure is proportional to the wavelength of the incidence sound.[22] Thus, it makes sense to divide β by the sound wavelength λ. Then, Eq. (1) becomes

$$\alpha = \alpha_0 + \mu\lambda E, \tag{8}$$

where $\mu = \beta/\lambda$ and Eq. (7) can be written as

$$\alpha_x = \alpha + \mu\lambda(E_x - E). \tag{9}$$

μ is often assumed to be equal to 0.25.

B. Thomasson's method

The edge effect causes the sample to be perceived as having a larger surface area than its actual size, commonly known as the virtual area. The factor K is expressed as the ratio between the virtual area S' and the actual area S of the sample, as depicted in the following equation:[12,15,16]

$$K = \frac{S'}{S} = 1 + \frac{\lambda E}{4}. \tag{10}$$

Then, the SAC of an infinite sample can be expressed as

$$\alpha_0 = \frac{\alpha}{K}. \tag{11}$$

Similarly, for a sample size whose SAC is to be predicted, the factor K_p can be written as

$$K_P = 1 + \frac{\lambda E_x}{4}. \tag{12}$$

The absorption coefficient of a material α_P with a size different from that measured in the reverberation room can be estimated using linear interpolation in α_0 between equating the α_0 values for the two different size samples when $\alpha_0 = 1$ and equating the SACs for the two different size samples when $\alpha_0 = 0$ using the following equation:

$$\alpha_P = \alpha_M \left[\alpha_0 \frac{K_P}{K_M} + (1 - \alpha_0) \right], \tag{13}$$

where K_M is calculated for the measured sample using Eq. (10) and α_M is the SAC of the measured sample. This method is based on one of Thomasson's proposed approaches.[15] While this method utilizes one-quarter wavelength for extending the virtual surface, it is worth noting that Thomasson's theory also presents a more intricate approach for calculating this extension width.

C. Geometric method

In the geometric method, the extra area is assumed to be the area outside the specimen which has a maximum distance of some value (usually one quarter of a wavelength) from the specimen in a direction normal to one of the edges of the rectangular specimen.[5,16,23] This means that the external squares at each of the four corners whose side length is the maximum distance are not included in the extra external area. The extra external area is considered to be the extra area from which the specimen can absorb sound due to the diffraction effect. However, the other interpretation is that the extra area is the area inside the specimen area which absorbs more sound energy due to the increase in sound pressure caused by diffraction. This extra area is the area inside the specimen which has a maximum distance of some value (usually one quarter of a wavelength) from the specimen edge in a direction normal to one of the edges of the rectangular specimen. In the case of Thomasson's method, this interpretation means that the areas of the squares at each of the four corners whose side length is the maximum distance is counted twice. There is an argument for counting the areas of these squares twice because they are within the maximum normal distance from two edges of the specimen. However, there is also an argument for not counting these areas twice and, in this case, it also makes sense not to allow the strips of extra area due to the opposite edges of the specimen to overlap. This approach limits the maximum extra inside area to be equal to the area of the specimen. Author R.A.H. adopted this approach and found that assuming that the width of the additional area was half of the wavelength worked well for the materials whose measured sound absorption he was using for testing. This is not surprising given the known variation of β between different materials. His equation for the extra virtual area of the measured sample is

$$T_m = 2a_m b_m - (a_m - 2d_m)(b_m - 2d'_m). \tag{14}$$

His equation for the extra virtual area of a sample whose SAC to be predicted is

$$T_p = 2a_p b_p - (a_p - 2d_p)\left(b_p - 2d'_p\right), \tag{15}$$

where

$$d_{m,p} = \begin{cases} \frac{1}{2} a_{m,p} & \text{for } a_{m,p} < \lambda, \\ \frac{1}{2}\lambda & \text{for } a_{m,p} > \lambda \end{cases} \tag{16}$$

and

$$d'_{m,p} = \begin{cases} \frac{1}{2} b_{m,p} & \text{for } b_{m,p} < \lambda, \\ \frac{1}{2}\lambda & \text{for } b_{m,p} > \lambda. \end{cases} \tag{17}$$

In the above equations, a and b are the side lengths of a sample. d is the width of strip with a length of a and d' is the width of strip with length of b. Subscript m denotes the measured sample and p denotes the predicted sample. Subsequently, the SAC of a different-sized sample can be estimated through measurement of a single-sized sample using the following equation:

$$\alpha_p = \alpha_m \frac{S_m T_p}{S_p T_m}, \tag{18}$$

where $S_m = a_m b_m$ and $S_p = a_p b_p$. This method called the first geometric method.

Like Thomasson, the author R.A.H. also found it necessary to carry out interpolation as a function of the SAC. Thus, the intermediate value of the SAC for the measured sample becomes

$$\alpha'_m = \alpha_m \frac{S_m}{T_m}. \tag{19}$$

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Then, the equation for the extra virtual area of the measured sample is

$$T_m{}' = S_m + (T_m - S_m)\alpha'_m. \tag{20}$$

The equation for the extra virtual area of a sample whose SAC to be predicted is

$$T_p{}' = S_p + (T_p - S_p)\alpha'_m. \tag{21}$$

Similarly, the second geometric method which uses linear interpolation is given by

$$\alpha_p{}' = \alpha_m \frac{S_m T_p{}'}{S_p T_m{}'}. \tag{22}$$

III. EXPERIMENTAL DATA ANALYSIS

This section investigates the phenomenon of the edge effect using experimental data and compares them with the theory. To characterize the edge effect of the sound absorbers, this research gathered all the published data on the edge effect from multiple sources. References 4, 6, and 10–12 directly provided the value of β but for Refs. 13, 15, 17, 24, and 25 it was necessary to calculate β by using linear regression on their provided SAC data for each frequency, following the procedure in Fig. 1. The samples used, their material information, the volume of the associated reverberation room, and the sources of the data are listed in Table I. The selected samples were installed as one area of material, and their edges were covered with a non-absorptive material. This study does not consider the measurement of separated pieces of material where there are gaps between each separated piece of material. The experimental results for samples H1 to H4 were measured in the Armstrong Acoustics Laboratory using the ASTM-C423 standard.[26] The reverberation room has a volume of 264.7 m^3 with total surface area of 255 m^2, and it is equipped with 8 diffuser panels, totalling a diffuser surface area of 64 m^2. It is noteworthy that samples from two sources, which have similar information regarding material type, thickness, and density, are believed to be of the same material, despite being obtained from different participating laboratories. Because of the varying specimen geometries, quantities of samples of various sizes, and diverse experimental results, those data are still valuable and deserving of analysis due to their diversity.

Figure 2(a) displays the edge effect relative parameter β as function of frequencies for all materials from Table I. At the first glance, the plot appears to be chaotic, but the most of data is distributed between 0.2 and 0 in the y axis direction. The trend of all curves exhibits an increase to a peak and then slowly decreases. To figure out the influence of the frequency on the edge effect, the average value of β for each one-third octave band centre frequency from all samples is depicted in Fig. 2(b), and the error bars represent the uncertainty of the 95% confidence intervals. There is a peak at 500 Hz, indicating that the edge effect is significant at this frequency for most materials. It seems likely that the material properties of the test specimen may affect the strength of the edge effect at different frequencies. However, the

TABLE I. The measurements from multiple sources that were analyzed.

Sample	Thickness [mm]	Density [kg/m^3]	Quantity of various sizes	Material type	Volume of reverberation room [m^3]	Source
H1	50.8	102.84	4	glass wool	264.7	experiment
H2	25.4	193.5	4	mineral wool	264.7	experiment
H3	25.4	102.84	4	glass wool	264.7	experiment
H4	25.4	193.5	4	mineral wool	264.7	experiment
Gomperts, M1	50	100	12	mineral wool	400	Ref. 4
Gomperts, M2	100	100	6	mineral wool	400	Ref. 4
Sauro, S1	25.4	102.84	9	glass wool	275	Ref. 17
Sauro, S2	50.8	102.84	4	glass wool	275	Ref. 17
Sauro, S3	50.8	102.84	4	glass wool	275	Ref. 17
Thomasson	50	50	3	mineral wool	200	Ref. 15
Hughes	50.8	6	5	ultralight foam	291.7	Ref. 13
Kawai	25	32	4	glass wool	317.4	Ref. 24
Bartel, M1	52	50	8	glass wool	425	Ref. 10
Bartel, M2	15	140	8	glass wool	425	Ref. 10
Bartel, M3	13	330	8	weed fiber	425	Ref. 10
Ten Wolde, A	50	100	10	mineral wool	199	Ref. 6
Ten Wolde, B	100	60	10	mineral wool	199	Ref. 6
Ten Wolde, C	38	90	10	straw fibre	199	Ref. 6
Ten Wolde, D	20	90	12	straw fibre	199	Ref. 6
Vercammen	100	44	unknown	mineral wool	214	Refs. 12,29
Dekker, M1	16	90	8	straw fibre	33.7	Ref. 11
Dekker, M2	40	60	7	glass wool	33.7	Ref. 11
Northwood	50	100	3	mineral wool	unknown	Ref. 25
Davern	50	100	3	mineral wool	607	Ref. 27

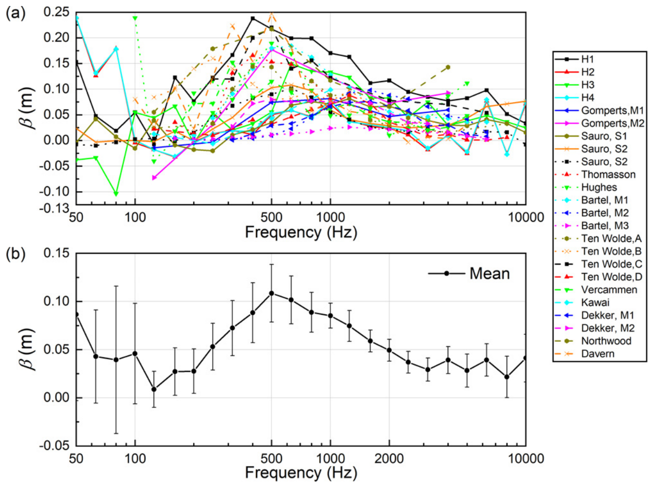

FIG. 2. (Color online) (a) The measured edge effect parameter β as a function of frequency for the materials listed in Table I. (b) The mean of all β as function of frequency. The error bars represent the 95% confidence intervals.

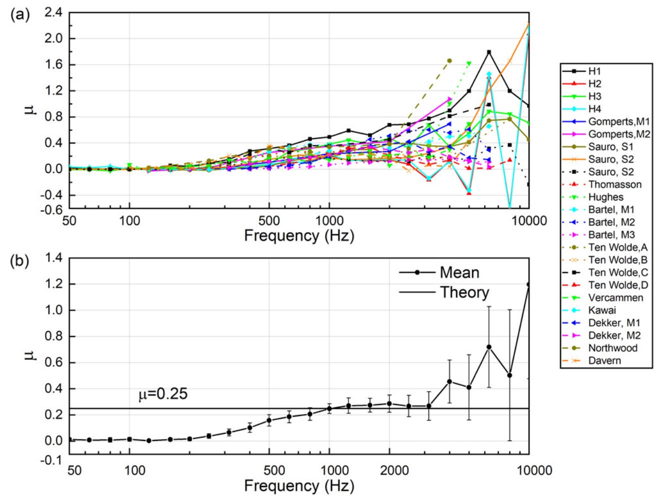

FIG. 3. (Color online) (a) The measured edge effect parameter μ as a function of frequency for the materials listed in Table I. (b) The mean of all μ as function of frequency compared with $\mu = 0.25$. The error bars represent the 95% confidence intervals. Note μ is equal to β divided by wavelength.

additional sound absorption of the materials basically occurs between 250 and 2000 Hz, which is consistent with the theory, since the difference between the specific acoustic susceptance of most materials and their surrounding surface area is lower in the low-frequency range. In the high-frequency range, the area of the specimen subject to the increased acoustic pressure is small because the wavelength of sound in air is small, and thus the edge effect is also small. However, there is a reasonable discrepancy in experimental data compared to the theory. The uncertainty of the experimental slope β in the low-frequency range[14] is large due to high measurement uncertainty of the SACs because of low statistical modal overlap of the reverberation rooms and the low SACs of most materials. In the high frequency range, the uncertainty of the experimental slope β is a result of the small differences between the measured SACs and moderate experimental uncertainty. Furthermore, the experimental uncertainty could be increased due to varying amounts of diffusivity between the different reverberation rooms, especially for highly sound absorbing samples.[1,27–29] In conclusion, the impact of the edge effect on the SAC of materials of varying sizes is found to be more significant at medium frequencies compared to high and low frequencies, regardless of whether the SAC values are obtained through experiment or through theoretical calculation.

Since β is a function of the wavelength, it makes sense to analyze the distribution of $\mu = \beta/\lambda$ for all samples. The data from Fig. 2(a) divided by the wavelength of the corresponding frequency is shown in Fig. 3(a). Most of the values of μ are in the range from 0 to 0.6. The data generally exhibit two types of curve trend. One is slowly increasing and then steady decreasing. The other is to rise and fall steadily like the first trend at the beginning, and then have larger fluctuations at the higher frequencies. Those curves with large oscillations may be caused by experimental uncertainty. As the frequency increases, parameter μ is increased due to the short wavelength. The mean of the data and the 95% confidence interval of the mean are demonstrated in Fig. 3(b). To compare with the theory, the line of $\mu = 0.25$ is also shown. In Fig. 3(b), μ is close to zero in the low frequency before increasing to be close to 0.25 in the mid-frequency range. Above 3 kHz the mean and the uncertainty increase. From 630 Hz to 3.15 kHz, the experimental mean value of μ is close to 0.25, which is consistent with the assumption that the width of additional area of the effective absorption surface is approximately equal to $\frac{1}{4}\lambda$. In the high-frequency range, significant fluctuations are observed, due to the uncertainty of measurement of the parameter μ when the sound wavelength is short. In conclusion, the quarter wavelength width of the additional apparent area of the specimen is not correct in the low frequency range. Based on a review of the measurement database, an improved method is proposed in Sec. IV.

IV. DEVELOPMENT OF THE EMPIRICAL EQUATION

To estimate the random incidence SAC with different sizes of the same material, a simple empirical equation was derived based on measurement data. Due to the influence of the material properties, the thickness t and the density ρ of each material were used. The transformation function for frequency f is

$$X = \frac{ft\left(\frac{\rho}{\rho'}\right)^{0.17}}{c} = \frac{t\left(\frac{\rho}{\rho'}\right)^{0.17}}{\lambda} \quad \text{with} \quad \rho' = 1\,\frac{\text{kg}}{\text{m}^3}, \qquad (23)$$

where ρ' represents the reference density used to normalize X into a dimensionless quantity and c is the speed of sound in air. The exponent 0.17 is applied to cluster the measurement data of materials, enhancing the suitability for curve-fitting optimization. The x axis variable has been changed from f to X, and the plot of the μ as a function of X is depicted in Fig. 4. Since the vertical scale (y axis) is

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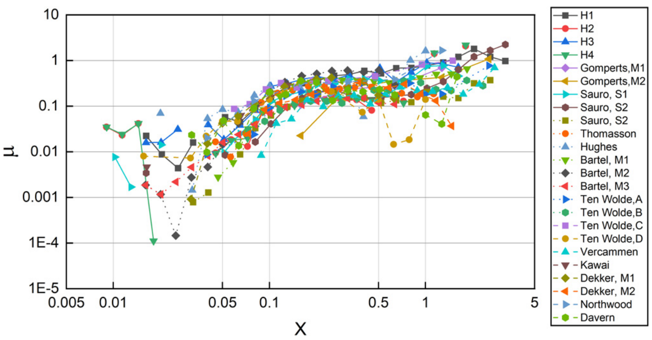

FIG. 4. (Color online) The edge effect parameter μ as a function of X for the materials of Table I.

logarithmic, the continuity of the data is interrupted by negative experimental values. One of the reasons for employing a transformation function is to address the substantial influence of material thickness. In this study, measurements were conducted on the same material with varying thicknesses (1, 2, and 4 in.). The observed peaks, resulting from the edge effect, demonstrated that doubling the thickness caused the peak absorption frequency to shift down by one octave for each doubling of thickness, while preserving the shape of the peak. Experimental data for different thicknesses of the same material indicated peak frequencies of approximately 1200–1250 Hz for the 1-in. sample, 600–650 Hz for the 2-in. sample, and 300–325 Hz for the 4-in. sample. By applying Eq. (23), the impact of thickness variation can be accounted for in the analysis.

According to the new figure, all the experimental data show the same trend, although there is a considerable spread of the data. Based on the shape of the curves and the high frequency theory, the following mathematical model was proposed:

$$\mu = \left[\left(C_1 X^{-C_2} \right)^{-C_4} + {C_3}^{-C_4} \right]^{-{C_4}^{-1}}. \tag{24}$$

The values of the constants C1–C4 were determined using optimization. Other models such as polynomial functions, Gaussian functions or other bell-shaped functions were also tried but did not work any better. A MATLAB code was developed to find the constants C1 to C4 that best fitted all the experimental data, using the nonlinear curve-fitting algorithm to minimize the mean square differences,[30]

$$\min_C \| F(C, xdata) - ydata \|_2^2 = \min_C \sum_i \left(F(C, xdata_i) - ydata_i \right)^2, \tag{25}$$

where F is objective function given by Eq. (10). The elements of the coefficient vector C are the parameters to be estimated and *xdata* is the input data such as the frequency, the speed of sound in air, the sample thickness, and the density. *ydata* are edge effect parameters μ derived from all the measured data. i is the number of data. In this paper, the total number of data points is 391. After data preprocessing, the number of data points is reduced to 352 due to the elimination of negative data points.

The four parameters, C1–C4, were determined using optimization (100.143, 2.835, 0.26, and 1.944, respectively). Hence μ and β could be predicted for any frequency, any thickness and density of material. Once the SAC of a material for one size of sample and its β are known, the SAC of any size samples of the same material can be estimated using Eq. (9). Inserting the values of the four constants into Eq. (24), the final empirical function is given by

$$\mu = \left(1.293 \times 10^{-4} X^{-5.51} + 13.71 \right)^{-0.5145}, \tag{26}$$

where this function is plotted together with the measured data from Fig. 4 in Fig. 5. It is worth mentioning that the data are well-fitted in the range of $0.05 < X < 0.5$, and most of the data are also clustered in this range. The increase in the SAC due to the edge effect is greatest in the mid-frequency range and for thicker materials, making the curve fitting in this range the most important. Outside this range, the data are scattered due to the uncertainties of the measurement mentioned in Sec. III. The empirical formula addresses the shortcomings of the quarter-wavelength approach. Specifically, it addresses two issues: (1) For frequencies below 1000 Hz, the value of μ deviates from 0.25. (2) The value of μ for the medium frequency range is only slightly above 0. 25 and thus is close to the theoretical value of 0.25. Furthermore, utilizing a straightforward empirical function is more convenient compared to intricate calculation using numerical integration[15] or approximate analytical formulas,[16] which requires an in-depth understanding of acoustics.

It is important to note that the empirical method is not flawless, as shown by the data distribution in Fig. 5. Even though the thickness and density of the material have been taken into account, the data remains dispersed. It might be possible to reduce the spread by using other physical or acoustical properties of the materials such as the imaginary parts of the specific acoustic admittances, the five or six parameters of the equivalent fluid model,[21] or the airflow resistivity used by the Delany-Bazley model.[31] However, obtaining those properties can be difficult and requires additional experimental testing. This also complicates the

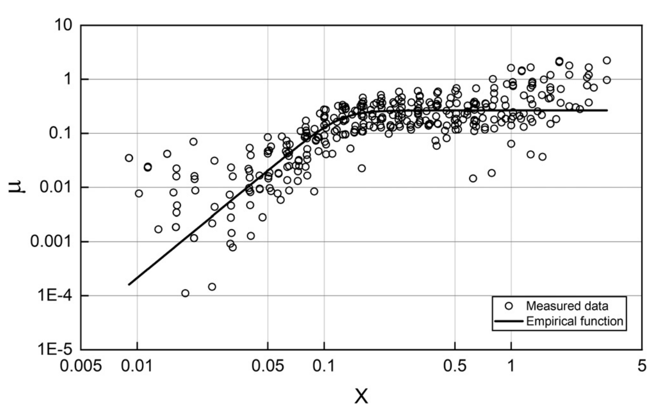

FIG. 5. The scatterplot of the measured data about the empirical function.

empirical model calculations and elevates the barrier for implementing this method, as users may not have access to the necessary parameters. By contrast, the thickness and density of a material are among the most easily determinable properties, and they have a great impact on its acoustical performance. The references in Table I also do not provide detailed material properties, with the exception of Northwood and Thomasson, which obstructs further research on the edge effect phenomenon and the empirical function. In summary, the proposed empirical function is currently the optimal solution. The next section compares the empirical method with other estimation methods to verify its feasibility.

V. ESTIMATION OF THE ABSORPTION COEFFICIENT OF AN ABSORBER FROM A DIFFERENT SIZE

To validate the empirical equation, it has been used to estimate the absorption coefficient of an absorber sample from measurements on a different size sample of the same material. Thomasson's method, the first and second geometric methods and the analytical method[16] have also been used to estimate the SAC from a different size sample. There are four versions of analytical method in Ref. 16 and this paper uses the most optimal one to compare with the other methods. The first geometric method follows Eq. (18), the second geometric method follows Eq. (22) and the method using the empirical function is given by Eqs. (9) and (26). Figure 6 illustrates the performance of the different methods for estimating the random incidence SAC for samples of H1, Sauro's S1, Thomasson, and Northwood. The dashed lines are the predicted data and the solid lines are the measured data. The figures on the left side present the results of predicting large size samples from small size samples, while the figures on the right side show the reverse. More specifically, a large sample size corresponds to a lower relative edge length, whereas a small sample size corresponds to a larger relative edge length. The results show that after size correction, the estimated values are consistent with the measured values for all approaches, except for a discrepancy for predicting S1 using the first geometric method. This is the reason why it is necessary to carry out interpolation for the geometric method. The empirical function performs well especially for predicting small size samples from large size samples.

To compare different approaches, the root mean square error (RMSE) has been used to evaluate the results for both direction (small to large or large to small). Due to the limited SAC data provided by the references, only 12 samples have been used to compare different approaches, which are shown in Table II. Without correction in the table represents the RMSE between the experimental data of small size sample and large size sample. The RMSE for no correction is larger for most materials, indicating that the measured SAC significantly depends on the sample's geometric properties. According to the mean of RMSE of all methods, it is shown that the empirical function is a reliable method to estimate the SAC from different size samples, which reduces the RMSE value by almost half compared to no correction. Thomasson's approach, the analytical method and the second geometric method are applicable to all materials, which effectively reduces the RMSE value. However, the first geometric method may not be reliable for some materials and may not be as effective for estimating small size samples from large size samples. The corrections from those methods significantly reduce the RMSE value for samples of H1, H3, Sauro's S1, Thomasson, Kawai, Northwood, and Davern. The RMSE value decreases by at least 50% and by up to 80% for these samples. The results for other samples also get better when using those estimation methods. It can be noted that the estimation results show greater significance for the samples that are impacted by a high degree of the edge effect, and those sound absorbing materials usually are thick or have high sound absorption.

The empirical function has a consistent RMSE value regardless of the direction of estimation. However, the other methods have higher errors when estimating a small size sample from large size sample than vice versa. Therefore, the empirical function is the most accurate method for estimating a small sample from a large sample among the five methods. The analytical method is the most accurate way for predicting a full-size sample from a small size sample, though this method is complicated. In addition, the second geometric method is recommended as an alternative to the analytical method, as it is less complex and easier to apply. Although Thomasson's method is the simplest to calculate, it is less competitive in terms of accuracy compared to other methods. The first geometric method may not always be reliable; it sometimes produces ridiculous results such as sample S1 and S3 but sometimes performs very well for predicting large size samples. In conclusion, the empirical method has proven to be useful for estimating the SAC of an absorber of a different size.

VI. CONCLUSION

This study investigated the SAC measurement results in a reverberation room. A new empirical function was introduced to predict the random incidence SAC of a sample based on SAC measurements on another size sample of the same material. The experimental data from 24 distinct materials obtained from multiple sources were analyzed to assess the impact of the edge effect. The results indicate that the edge effect significantly influences the mid-frequency range (250 Hz to 2 kHz) and that the width of the apparent additional area of the sound absorber is approximately equal to a quarter wavelength within this range. The empirical function was derived from these measurements and the linear relationship between the SAC and the relative edge length, and the implementation of the empirical function only requires the values of the material thickness and the density. Thomasson's method, the two geometric methods, and the analytical method were used and compared with the empirical method. The results demonstrate that each approach has its strengths for estimating the SAC of samples of different

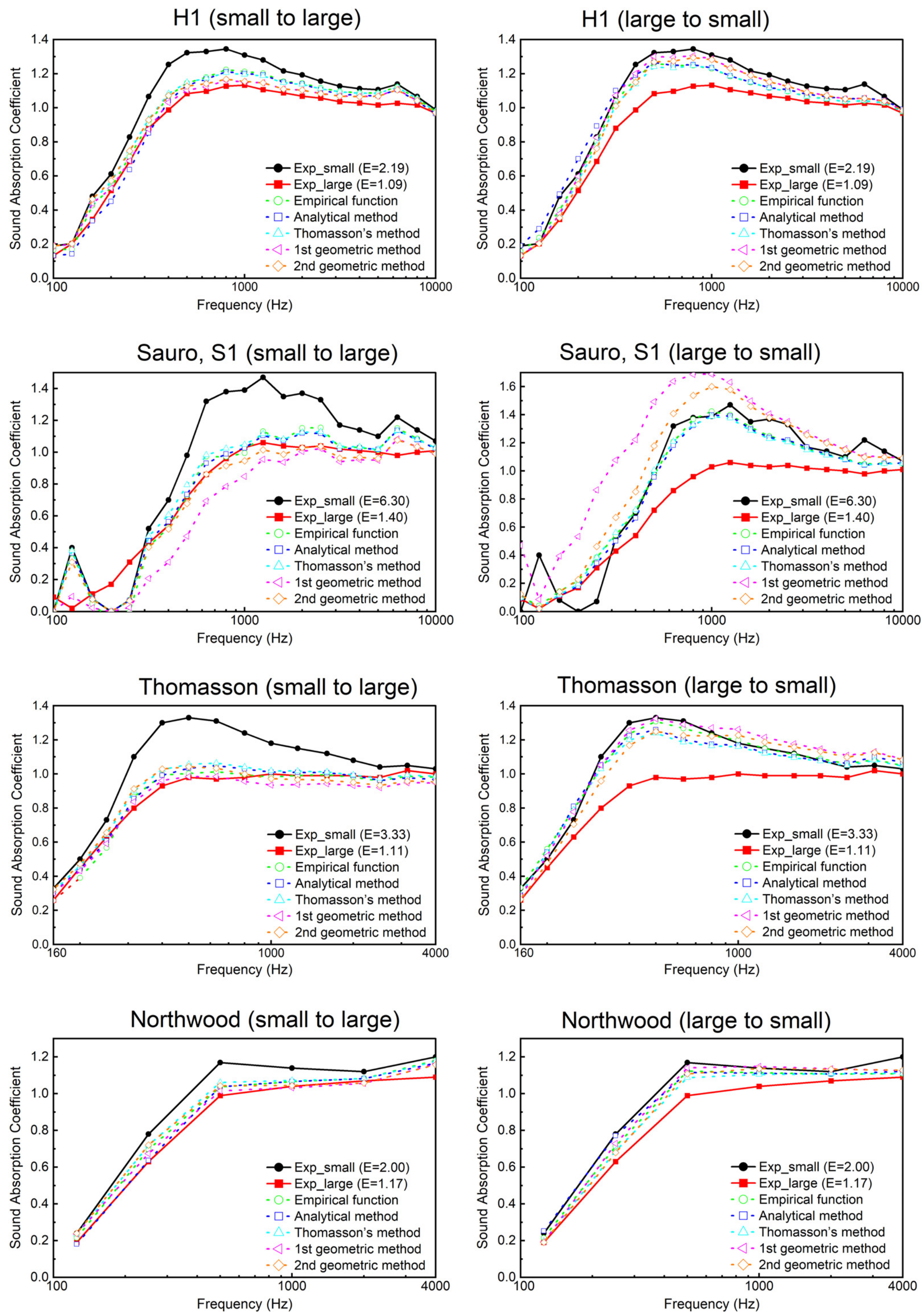

FIG. 6. (Color online) Estimation of the SACs from another size sample of the same material using five different methods.

sizes. Among the five approaches, the empirical function performed best for predicting small size samples from large size samples. The great advantage of the empirical method is that it produces the same errors in each direction. The analytical method was found to be the most accurate approach for estimating full size samples from small size samples. However, this method involves complex formulas and requires a thorough understanding of acoustics. The geometric method which uses interpolation as a function of the SAC is also recommended as a simpler alternative to the analytical method, while still maintaining a reasonable level of accuracy. The proposed empirical function offers a new solution for estimating the random incidence SAC of samples with different sizes of the same material by applying the edge effect theory in practice. This research is valuable for estimating the SAC of a room. SAC data provided by

TABLE II. The comparison of the RMSE between the experimental and predicted SAC for different approaches across the applicable frequency range (Unit: 10^{-2}).

Sample	Target size	Empirical function	Thomasson's method	Analytical method	1st geometric method	2nd geometric method	Without correction
H1	small to large	6.52	6.59	5.57	3.99	5.14	14.8
	large to small	6.52	7.29	6.33	4.30	5.75	14.8
H2	small to large	3.53	3.06	2.96	4.24	3.20	4.20
	large to small	3.53	3.12	3.09	4.62	3.42	4.20
H3	small to large	4.06	4.89	3.51	3.04	3.72	8.07
	large to small	4.06	5.24	4.07	3.35	4.02	8.07
H4	small to large	3.79	3.13	3.00	4.28	3.26	4.35
	large to small	3.79	3.19	3.15	4.66	3.50	4.35
Sauro, S1	small to large	13.4	12.2	11.4	14.4	10.1	26.3
	large to small	13.4	13.8	13.2	34.8	16.7	26.3
Sauro, S2	small to large	11.5	11.6	11.3	11.5	11.1	13.0
	large to small	11.5	11.8	11.5	12.1	11.4	13.0
Sauro, S3	small to large	7.35	7.18	7.02	8.70	7.28	8.10
	large to small	7.35	7.35	7.23	10.2	7.78	8.10
Thomasson	small to large	3.72	5.35	4.09	4.56	5.64	20.6
	large to small	3.72	6.02	5.46	5.39	7.14	20.6
Hughes	small to large	6.21	5.77	6.27	5.39	5.74	8.72
	large to small	6.21	6.03	6.73	5.54	5.92	8.72
Kawai	small to large	6.60	7.69	6.50	2.63	5.05	17.6
	large to small	6.60	8.74	8.17	3.61	6.01	17.6
Northwood	small to large	5.22	6.27	3.90	4.12	5.49	11.7
	large to small	5.22	6.67	4.14	4.38	6.02	11.7
Davern	small to large	1.73	2.00	1.95	2.36	2.12	4.35
	large to small	1.73	2.08	1.99	2.53	2.28	4.35
Mean	small to large	6.14	6.31	5.62	5.77	5.65	11.8
	large to small	6.14	6.78	6.26	7.96	6.66	11.8

manufacturers are measured in standard dimensions, but the surfaces in a room that contribute to sound absorption vary in size and shape, making it difficult to estimate the SAC accurately. The empirical function method can be used to obtain a more precise SAC for a room's specific surfaces, resulting in a more accurate estimate of the room's SAC. This study could be expanded by employing alternative approaches such as machine learning techniques,[32] provided that sufficient measurement data were available.

AUTHOR DECLARATIONS

Conflict of Interest

The authors state that they have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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